

## PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

## MENTAL CALCULATIONS

These are a selection of mental calculation strategies:
See NNS Framework Section 5, pages 52-57 and Section 6, pages 58-65

## Doubling and halving

Applying the knowledge of doubles and halves to known facts.
E.g. $8 \times 4$ is double $4 \times 4$

Knowing that doubling is the same as multiplying by 2 .

## Using multiplication facts

Tables should be taught from Y1 summer term onwards, either as part of the mental oral starter or other times as appropriate within the day.
Year 210 times table 5 times table 2 times table

Year 34 times table 8 times table 3 times table 6 times table 9 times table

Year $4 \quad 7$ times table 11 times table 12 times table Derive and recall all multiplication and division facts up to $12 \times 12$

Years 5 \& 6 Derive and recall quickly all multiplication and division facts up to $12 \times 12$. Work out products such as $70 \times 5,70 \times 50$ and $700 \times 50$ using the related fact $7 \times 5$ and their knowledge of place value.

## Using and applying division facts

Children should be able to utilise their knowledge of multiplication to derive other facts.
E.g. If I know $3 \times 7=21$, what else do 1 know?
$30 \times 7=210,300 \times 7=2100,3000 \times 7=21000,0.3 \times 7=2.1$ etc
Use closely related facts already known

$$
\begin{aligned}
13 \times 11 & =(13 \times 10)+(13 \times 1) \\
& =130+13 \\
& =143
\end{aligned}
$$

Multiplying by 10 or 100
Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left and the addition of 1 place value holder.
Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left and the addition of 2 place value holders.

## Partitioning

$$
\begin{aligned}
23 \times 4 & =(20 \times 4)+(3 \times 4) \\
& =80+12 \\
& =102
\end{aligned}
$$

Use of factors
$8 \times 12=8 \times 4 \times 3$
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

Children should not be made to go onto the next phase if:

1) They are not ready.
2) They are not confident.

Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to consider if a mental calculation would be appropriate before a written method.

## Multiplication - Year 1 Repeated Addition

## Concrete:



## Pictorial:



Children need to make and draw the image they are representing in lots of different ways to be able gain a good enough
understanding. Resources like paper bowls, tissue shapes, beads, compare bears etc.... are all really useful when teaching this concept.

[^0]Although the answer is the same, be sure about the ways that you teacher multiplication. $2 \times 5$ is 2 five times. $5 \times 2$ is 5 two times. Ensure that teachers and TAs understand the difference between these as the representations will look different.

## Multiplication - Year 1

Using a number line to show repeated groups.

## Concrete:



Concrete objects to show repeated groups. Both of these representations show $10 \times 3$ ( 10 three times).
Children should use these when they know have a concept of unitising. (Knowing the object represents 10 and that isn't going to change.)

## Pictorial:



## Abstract: (10 x 3)



Children might need to begin using a number line with numbers on it, but when they are more confident with the numbers they are counting in, should be able to jump on without the numbers being

Sarah has $\qquad$ socks altogether.

## Multiplication - Year 2 <br> Using Arrays

## Concrete:

Use arrays to illustrate commutativity counters and other objects can also be used.
$2 \times 5=5 \times 2$


## Pictorial:

Children to represent the arrays pictorially.

OR
$5 \times 2$

Until children can articulate what multiplication is, it is best not to talk about commutativity so that they don't get confused.


Abstract:

Children to be able to use an array to write a range of calculations e.g.
$10=2 \times 5$
$5 \times 2=10$
$2+2+2+2+2=10$
$10=5+5$

Children will need to complete this activity having made or drawn an array. The more 'facts' they can write about an array, he greater their understanding of them.

## Multiplication - Year 3

Multiplying by partitioning
Concrete: ( $15 \times 4$ )

$40+20=60$

Using base 10 materials to multiply $15 \times 4$.

Children multiply the ones and they multiply the 10s and add the two parts together. Using the place value grid helps children when they move onto the pictorial and abstract steps.

Pictorial:


Concrete:


Children need to show all of the steps that they take to be able to find a solution.
Children need to be encouraged to use their times tables facts when working with partitioned numbers where possible.

## Multiplication - Year 3

## Step 1 - formal multiplication method

## Concrete:

$23 \times 3=$


Using base 10 or place value counters (when children's understanding of place value is good), children partition the number they are multiplying and recombine. Ensure that children are using a place value grid and encourage talk about the value of the digits.

## Pictorial:

Children to represent the digits pictorially

6
9

Children can use the base 10 images rather than counters if they prefer. Continue questioning children about the value of the digits they are representing.

## Abstract:

| 23 | Expanded column multiplication. Ensure <br> that children write down each step of the <br> calculation and continue to talk about the <br> value of each digit. Teach this step <br> alongside the concrete or pictorial step so <br> children can see the size of the numbers <br> they are using. |
| :--- | :--- |
| $60(3 \times 3)$ |  |

## Multiplication - Year 4 <br> Formal written multiplication

Concrete: $23 \times 6=$


As the children are having to exchange here, they need to physically take the counters away and change it for the 10 or 100. This step is really important so ensure that children can articulate what they are doing and why.

Children to represent the base 10 (counters) pictorially eg:


Children to circle the counters they are exchanging and draw the new one on. They might want to circle in a different colour so that it is clear to them what they are doing. Encourage children to articulate what they are doing and why.

## Abstract:

$6 \times 23=$
23
$\times 6$
138
11

> Refer back to when you were teaching 'carrying' in addition and where you put the numbers to be carried. Be consistent in your approach to where they go in the written calculation.

## Multiplication - Year 5 and beyond Multiplying larger numbers.

## Concrete and Pictorial:

Children should be confident enough with the previous steps of multiplication that they don't need to use concrete or pictorial methods. If they aren't, please go back a stage until they are.

## Abstract: <br> Option 1:



The main piece of learning in this example is where the children put the numbers they are carrying for each step of the calculation. In this example, children only need to carry when multiplying by the 1s. In the example below, they have to carry when multiplying by the 10s as well. When children are first learning this, they might find it easier to use 2 different colours when multiplying and then add them at the end. Ensure that you are consistent in your approach across your school.

Carrying 'twice':

124
$\times 36$


## Using Conceptual variation:

## Different ways to ask $6 \times 23$

## Bar model:

David earns £23 each day for 6 days. How much does he earn?


David earns $\qquad$ altogether.

## Word problems:

Maisy had to swim 23 lengths, 6 times a week. How many lengths did she swim in 1 week?

With counters, prove that $6 \times 23=138$

## Procedural variation:

Find the product of 6 and 23
$6 \times 23=$
「-7 $=6 \times 23$


Conceptual variation:
What is the calculation?
What is the product?


## Scaling questions:

## Multiplication as scaling: Measures

These are simply problems like, "it take me 5 minutes to walk to school, it takes my friend 3 times as long. How long does it take her?"

Contexts will very often be measures. However, saying things like " 5 apples in a bag, I buy 1.2 bags" doesn't make sense in the real world.

- Year 3 National Curriculum objective (multiplication and division):
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems


## Scaling linked to the Bar Model

There is a polar bear that is three times the weight of a baby elephant. Altogether they weigh 700 kg . What is the weight of the polar bear?



[^0]:    Abstract:
    $2 \times 5=$
    $2+2+2+2+2=$

